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Abstract
It is shown that a microscopic system consisting of Brownian particles moving in a spatially asymmetric but periodic potential (ratchet) and contacting with the alternating hot and cold reservoirs along space coordinate and an external force applying on the particles may work as a refrigerator. In order to clarify the underlying physical pictures of the system, the heat flows via both the potential energy and the kinetic energy of the particles are considered simultaneously. Based on an Arrhenius' factor describing the forward and backward particle currents, expressions for some important performance parameters of the refrigerator, such as the coefficient of performance, cooling rate and power input, are derived analytically. The maximum coefficient of performance and cooling rate are numerically calculated for some given parameters. The influence of the main parameters such as the external force, barrier height of the potential, asymmetry of the potential and temperature ratio of the heat reservoirs on the performance of the Brownian refrigerator is discussed. The optimum criteria of some characteristic parameters are given. It is found that the Brownian refrigerator may be controlled to operate in different regions through the choice of several parameters.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Recently, there has been much interest in the study of microscopic engines and heat pumps (or refrigerators). One aspect is the need to have such devices in order to utilize energy resources available at the microscopic scale and the other aspect is the trend in miniaturization of devices demanding tiny engines and heat pumps that operate at the same scale. The molecular heat engines and heat pumps have been proposed as a class of microscopic machines based on recent developments in nanotechnology and single-molecule manipulations [1–7]. Many important results relative to the microscopic heat engines and heat pumps have been obtained [5, 8–16], but the investigation into the microscopic refrigerators is still at a preliminary stage. Thus, modeling microscopic refrigerators and finding how well they perform is a primary task to be undertaken at present.

The idea that Brownian microscopic heat engines can work under nonuniform temperature was first presented by Buttiker [17], van Kampen [18] and Landuaer [19]. Due to the importance of Brownian ratchets (motors) in developing molecular motors [1, 2], many researchers [3, 16, 20–24] have investigated the performance characteristics of the Brownian heat engines driven by a contact with the reservoirs at different temperatures and have obtained many meaningful conclusions. On the other hand, the refrigeration can be achieved by reducing the thermal noise of nanodevices with a feedback system that detects their velocities and applies on them a corresponding friction-like control force [25]. Van den Broeck and Kawai gave a novel model of the Brownian microscopic refrigerator consisting of two heat reservoirs with different temperatures in the presence of a constant force on a nanodevice [5]. Kim and Qian extended fluctuation theorems to a molecular system consisting of Brownian particles in a heat bath under feedback control of their velocities [7]. Van den Broek and Van den Broeck presented a chiral rotational model and studied the performance characteristics of the chiral molecular device operation as a heat engine or a heat pump [26, 27]. However, so far the optimal performance of irreversible Brownian refrigerators and their parametric optimum criteria have been rarely investigated. Thus, it is of great significance to study the optimal performance and parameters of Brownian refrigerators.

In this paper, we will give a detailed study of an irreversible Brownian micro-refrigerator in a spatially periodic temperature field. In the Brownian micro-refrigerator, the heat flows via the potential and kinetic energies of Brownian particles will be considered. The general expressions of several important parameters of the refrigerator, such as the coefficient of performance (COP), cooling rate and power input are derived. Moreover, the influence of some main parameters on the performance of the Brownian micro-refrigerator is analyzed in detail. Several novel results, which can reveal the general performance characteristics of Brownian micro-refrigerators, are obtained. The results obtained here will be helpful for deeply understanding the performance characteristics of Brownian micro-refrigerators.

2. An irreversible Brownian micro-refrigerator

The Brownian refrigeration system consists of two parts, Brownian particles which move along a spatially asymmetric but periodic structure potential (ratchet) and two heat reservoirs C and H at temperatures $\beta_C = (k_B T_C)^{-1}$ and $\beta_H = (k_B T_H)^{-1}$, respectively, where k_B is the Boltzmann constant. The particles are periodically in contact with the two heat reservoirs along space coordinate [3, 17, 21] and an external force F is applied to the particles, as shown in figure 1, where N^+ and N^- are, respectively, the numbers of forward and backward jumps per unit time, D ($0 < D < 1$) is a constant, L is the period length of the potential and U_0 is the barrier height of the potential. It is assumed that the rates of both forward and backward jumps
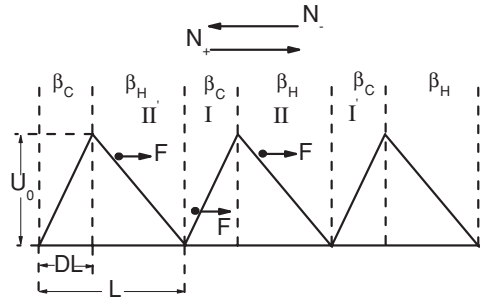
Figure 1. The schematic diagram of a thermally driven Brownian refrigerator.

are proportional to the corresponding Arrhenius' factor [23] and the system is in a state of stable flow, so that the numbers of forward and backward jumps per unit time are, respectively, determined by

$$\dot{N}_+ = \frac{1}{t} \exp[-(U_0 - FDL)/(k_B T_C)] \tag{1}$$

and

$$\dot{N}_- = \frac{1}{t} \exp\{-[U_0 + F(1 - D)L]/(k_B T_H)\}, \tag{2}$$

where t is a proportionality constant with a time dimension.

If $N_+ > N_-$, the thermal motor (ratchet) works as a two-reservoir micro-refrigerator. The refrigerator will absorb the heat from the cold reservoir at temperature T_C and release the heat to the hot reservoir at temperature T_H. When Brownian particles move in different regions, the change of the potential energy will result in a heat exchange between the refrigerator and the heat reservoirs. The heat flows from the cold reservoir to the refrigerator and from the refrigerator to the hot reservoir via potential are, respectively, given by

$$\dot{Q}_C^{\text{pot}} = (\dot{N}_+ - \dot{N}_-)(U_0 - FDL) \tag{3}$$

and

$$\dot{Q}_H^{\text{pot}} = (\dot{N}_+ - \dot{N}_-)[U_0 + F(1 - D)L]. \tag{4}$$

The heat flow resulting from the change of the kinetic energy of Brownian particles is much more complicated. When the particle lies in a region, it is in equilibrium with a heat reservoir. According to the theory of the energy equipartition, the average kinetic energy per particle is equal to $k_B T/2$. In order to keep the Brownian refrigerator operating continuously and stably, when the particles in region I leave this region, the particles with equal quantity must be supplied from the neighbor regions II′ and II, so that the number of particles in a region is constant. Consequently, when the particles leave regions II′ and II and enter region I, they will release the heat energy $(\dot{N}^+ + \dot{N}^-)k_B(T_H - T_C)/2$ to the cold reservoir to reduce their average kinetic energy. Similarly, when the particles leave regions I and I′ and enter region II, they will absorb the heat energy $(\dot{N}^+ + \dot{N}^-)k_B(T_H - T_C)/2$ from the hot reservoir to rise their average kinetic energy. It can be seen from the above analysis that the total heat flow from the hot reservoir to the refrigerator due to the change of the kinetic energy of Brownian particles, $\dot{Q}_H^{\text{kin}}$, is equal to that from the refrigerator to the cold reservoir, $\dot{Q}_C^{\text{kin}}$, i.e.,

$$\dot{Q}_H^{\text{kin}} = \dot{Q}_C^{\text{kin}} = (\dot{N}^+ + \dot{N}^-)k_B(T_H - T_c)/2 \equiv \dot{Q}^{\text{kin}}. \tag{5}$$

It is seen from equation (5) that the energy $(\dot{N}^+ + \dot{N}^-)k_B(T_H - T_C)/2$ is transferred completely from the hot reservoir to the cold reservoir. It causes the heat leak between the hot and cold reservoirs. This indicates the inherently irreversible nature of this heat flow.
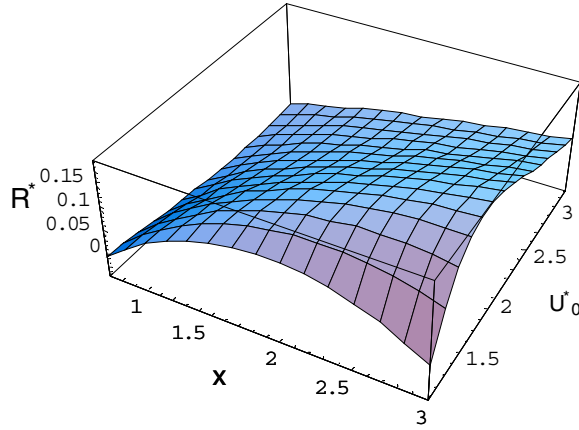
Figure 2. The variation of the cooling rate R^* with respect to the parameters x and U_0^*. The graph is presented for the parameters $\tau = 0.7$ and $D = 0.3$.

It is found from the above results that the total heat flows absorbed from the cold reservoir and released to the hot reservoir are, respectively, given by

$$\dot{Q}_C = \dot{Q}_C^{\text{pot}} - \dot{Q}_C^{\text{kin}} = (\dot{N}_+ - \dot{N}_-)(U_0 - FDL) - (\dot{N}_+ + \dot{N}_-)k_B(T_H - T_C)/2 \tag{6}$$

and

$$\dot{Q}_H = \dot{Q}_H^{\text{pot}} - \dot{Q}_H^{\text{kin}} = (\dot{N}_+ - \dot{N}_-)[U_0 + F(1 - D)L] - (\dot{N}_+ + \dot{N}_-)k_B(T_H - T_C)/2. \tag{7}$$

Thus, the coefficient of performance (COP), the cooling rate and power input of the irreversible Brownian refrigerator can be, respectively, expressed as

$$\text{COP} = \frac{\dot{Q}_C}{\dot{Q}_H - \dot{Q}_C} = \frac{U_0^*}{x} - D - \frac{(1 - \tau)}{2x} \frac{\mathrm{e}^{U_0^*(1-1/\tau)+[1+D(1/\tau-1)]x} + 1}{\mathrm{e}^{U_0^*(1-1/\tau)+[1+D(1/\tau-1)]x} - 1}, \tag{8}$$

$$R = \frac{k_B T_H}{t}\left\{(U_0^* - Dx)\left[\mathrm{e}^{-(U_0^*-Dx)/\tau} - \mathrm{e}^{-(U_0^*+(1-D)x)}\right]\right.$$
$$\left. - \frac{1}{2}(1 - \tau)\left[\mathrm{e}^{-(U_0^*-Dx)/\tau} + \mathrm{e}^{-(U_0^*+(1-D)x)}\right]\right\} \tag{9}$$

and

$$P = (\dot{Q}_H - \dot{Q}_C) = \frac{k_B T_H}{t}\left[\mathrm{e}^{-(U_0^*/\tau-Dx/\tau)} - \mathrm{e}^{-[U_0^*+(1-D)x]}\right]x, \tag{10}$$

where $\tau = T_C/T_H$, $D = L_1/L$, $U_0^* = U_0/(k_B T_H)$ and $x = FL/(k_B T_H)$. It can clearly be seen from equations (8)–(10) that the heat flow due to the change of the kinetic energy of Brownian particles affects the COP and cooling rate of the Brownian refrigerator, but it does not affect the power input of the Brownian refrigerator. In order to discuss conveniently, equations (9) and (10) can be rewritten in a dimensionless form, i.e., $R^* = Rt/(k_B T_H)$ and $P^* = Pt/(k_B T_H)$.

3. General performance characteristics of the micro-refrigerator

Using equation (9), one can plot a three-dimensional diagram (x, U_0^*, R^*) for given τ and D, as shown in figure 2, where the parameters $\tau = 0.7$ and $D = 0.3$ are chosen. It can be seen
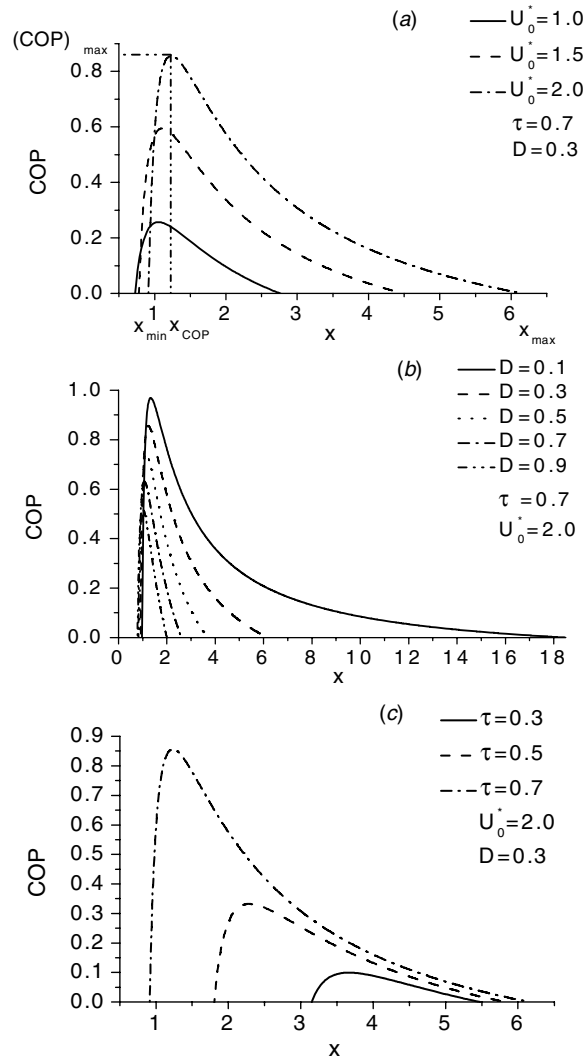
Figure 3. The curves of the COP varying with the dimensionless external force x for some given parameters: (*a*) $\tau = 0.7$ and $D = 0.3$, (*b*) $\tau = 0.7$ and $U_0^* = 2.0$ and (*c*) $U_0^* = 2.0$ and $D = 0.3$.

from figure 2 that the cooling rate R first increases and then decreases as x or U_0^* is increased. It clearly shows that there are local optimal values of x or U_0^* at which the cooling rate R attains its local maximum value for a given set of operating parameters. Similarly, using equation (8), one can plot a three-dimensional diagram (x, U_0^*, COP) for given τ and D. Its shape is similar to figure 2. Consequently, there are local optimal values of x or U_0^* at which the COP attains its local maximum value for a given set of operating parameters.

Using equations (8) and (9), one can generate the curves of the COP and cooling rate varying with the parameter x for given U_0^*, τ and D, as shown in figures 3 and 4. It can be seen from figures 3 and 4 that both COP and R^* are of concave curves that vanish at $x = x_{\min}$ and $x = x_{\max}$, and there exists a local maximum coefficient of performance $(\text{COP})_{\max}$ and a local maximum R^* in the region of $x_{\min} < x < x_{\max}$. Obviously, the maximum COP and R^* depend on the parameters U^*, τ and D.
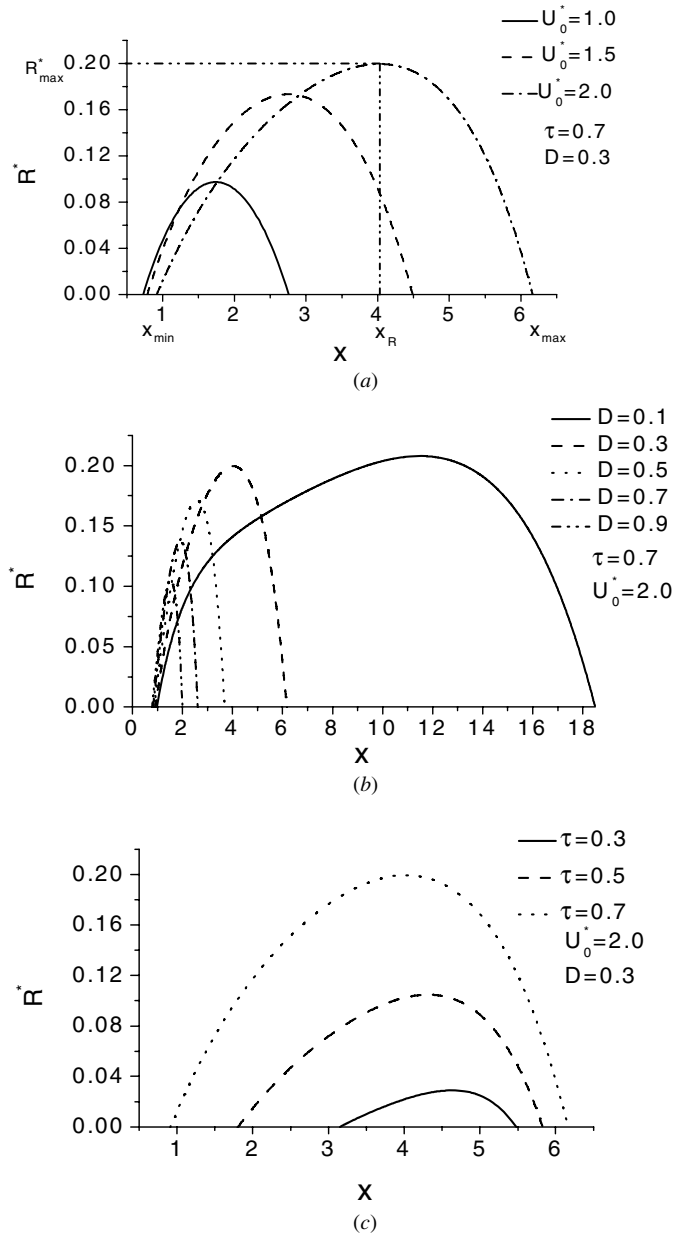
Figure 4. The curves of the cooling rate R^* varying with the dimensionless external force x for some given parameters. The values of the parameters τ, D and U_0^* are the same as those used in figure 3.

It can also be seen from figures 3 and 4 that for differently given parameters, $(COP)_{max}$, x_{COP} at the $(COP)_{max}$, R^*_{max}, x_R at the maximum cooling rate, x_{min}, and x_{max} will be different. For example, for given D and τ, the larger the U_0^*, the larger the x_{min}, x_{max}, x_{COP} and x_R; for given U_0^* and τ, the larger the D, the less the x_{min}, x_{max}, $(COP)_{max}$, x_{COP}, R^*_{max} and x_R; for given U_0^* and D, the larger the τ, the less the x_{min}, x_{COP} and x_R, while the larger the x_{max},
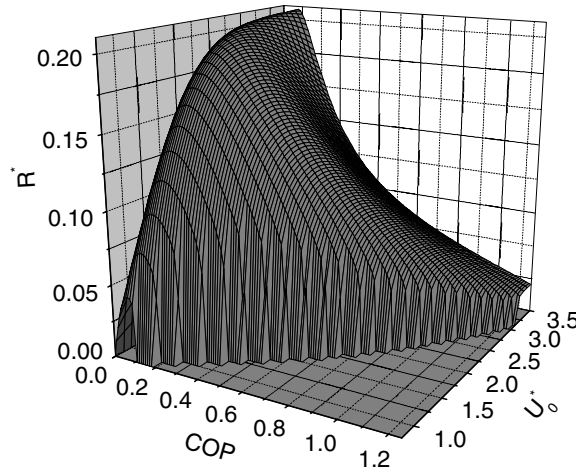
Figure 5. The variation of the cooling rate R^* with respect to the COP and dimensionless barrier height of the potential U_0^*, where the parameters $D = 0.3$ and $\tau = 0.7$ are chosen.

$(\text{COP})_{\text{max}}$ and R^*_{max}. Their physical meanings are very clear. When τ is increased, the power input required by the system will decrease, and consequently, both the cooling rate R and the COP will increase. If τ approached the unity, the system would get the thermal equilibrium while both the cooling rate R and the COP would increase quickly. It can be found from figure 1 and equations (6) and (7) that $U_0 - FDL$ is the energy required by a particle for a forward jump, while $U_0 + F(1 - D)L$ is the energy required by a particle for a backward jump. The larger the D, the less the $\dot{Q}_C$. Thus, when D is increased, both R and COP will decrease. It can also be found from equations (1), (2) and (7) that when U_0 is increased, $\dot{N}_+ - \dot{N}_-$ will decrease and $\dot{Q}_C$ will first increase and then decrease. Thus, both R and COP first increase and then decrease as U_0^* is increased.

In order to understand further the general performance characteristics of the irreversible Brownian refrigerator, equations (8) and (9) can be used to plot a three-dimensional graph (COP, U_0^*, R^*) and the cooling rate versus COP curves, as shown in figures 5 and 6. It is clearly seen that each curve in figure 6 is of a typical loop form of real conventional refrigerators [28, 29]. When the Brownian refrigerator is operated in the region of $x \leqslant x_{\text{COP}}$ or $x \geqslant x_R$, the cooling rate will decrease as the COP decreases. When the Brownian refrigerator is operated in the region of $x_{\text{COP}} \leqslant x \leqslant x_R$, the cooling rate will increase as the COP decreases and vice versa. It is thus clear that when

$$x_{\text{COP}} \leqslant x \leqslant x_R \tag{11}$$

is satisfied, the optimal regions of the cooling rate and the COP should be subject to

$$R^*_m \leqslant R^* \leqslant R^*_{\text{max}} \tag{12}$$

and

$$(\text{COP})_m \leqslant \text{COP} \leqslant (\text{COP})_{\text{max}}, \tag{13}$$

where R^*_m and $(\text{COP})_m$ are, respectively, the dimensionless cooling rate at the $(\text{COP})_{\text{max}}$ and the COP at the maximum cooling rate. Through the choice of the parameters x, U_0^*, D and τ, the Brownian refrigerator may be controlled to operate in different behavior regimes.

Similarly, for given x, τ and D, using equations (8) and (9), one can plot the curves of the COP and cooling rate varying with the parameter U_0^*, as shown in figures 7 and 8, where
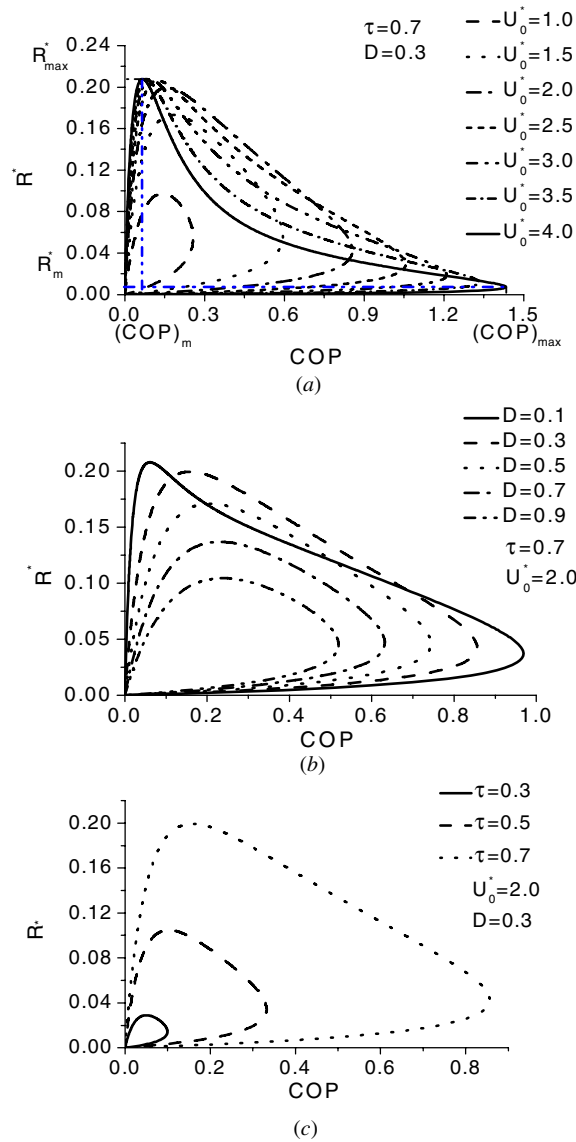
Figure 6. The curves of the cooling rate R^* varying with the COP for some given parameters. The values of the parameters τ, D and U_0^* are the same as those used in figure 3.

$U_{0\,\text{COP}}^*$ is the value of U_0^* at the $(\text{COP})_{\text{max}}$, U_{0R}^* is the value of U_0^* at the local maximum cooling rate, $U_{0\,\text{min}}^*$ and $U_{0\,\text{max}}^*$ are, respectively, the minimum and maximum of U_0^* at which the COP or the cooling rate is equal to zero. In figures 7 and 8, the parameters $\tau = 0.4$, 0.5, 0.6 or 07, $D = 0.1$, 0.3, 0.5, 0.7 or 0.9 and $x = 1.0$, 1.5, 2.0, 2.5 or 3.0 are chosen. It is seen from figures 7 and 8 that the COP or the cooling rate first increases and then decreases as U_0^* is increased. It clearly shows that there is an optimal value $U_{0\,\text{COP}}^*$ or U_{0R}^* at which the COP or the cooling rate attains its local maximum value for a given set of operating parameters. When the same values of the related parameters used in figure 7 are chosen, by means of numerical
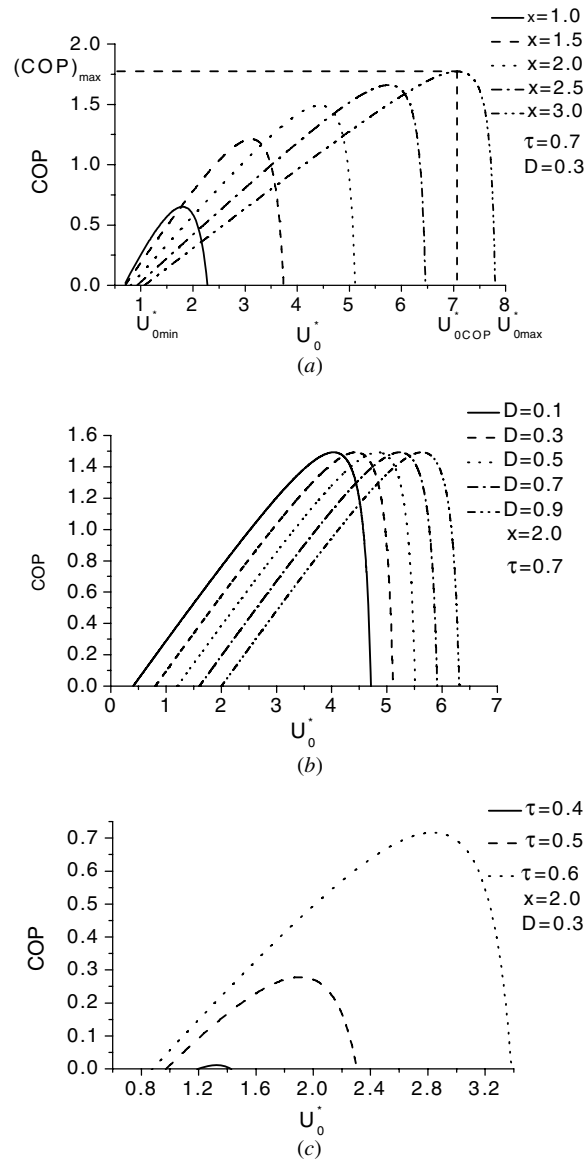
Figure 7. The curves of the COP varying with the dimensionless barrier height of the potential U_0^* for some given parameters: (*a*) $\tau = 0.7$ and $D = 0.3$, (*b*) $\tau = 0.7$ and $x = 2.0$, and (*c*) $x = 2.0$ and $D = 0.3$.

calculation, we can obtain the values of $U_{0\min}^*$, $U_{0\max}^*$, $(\mathrm{COP})_{\max}$, $U_{0\,\mathrm{COP}}^*$, $R_{\max}^*$ and U_{0R}^* for different operating parameters. It is seen from figures 7 and 8 that for given D and τ, the larger the x, the larger the $U_{0\min}^*$, $U_{0\max}$, $U_{0\,\mathrm{COP}}^*$ and U_{0R}^*; for given x and τ, the larger the D, the larger the $U_{0\min}^*$, $U_{0\max}^*$, $U_{0\,\mathrm{COP}}^*$ and U_{0R}^*, while $(\mathrm{COP})_{\max}$ and $R_{\max}^*$ do not vary with the parameter D; for given x and D, the larger the τ, the larger the $U_{0\max}^*$, $U_{0\,\mathrm{COP}}^*$, $(\mathrm{COP})_{\max}$, U_{0R}^* and $R_{\max}^*$, while the less the $U_{0\min}^*$. It is also seen from figures 7 and 8 that the performance characteristics between the cooling rate and the COP of an irreversible Brownian refrigerator
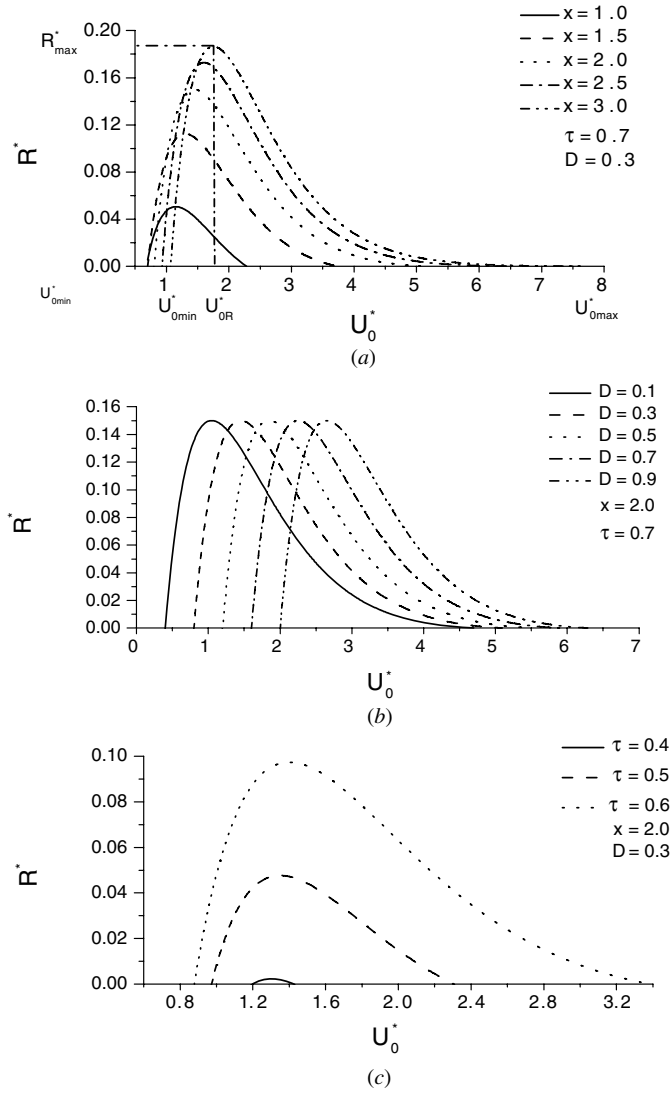
Figure 8. The curves of the cooling rate R^* varying with the dimensionless barrier height of the potential U_0^* for some given parameters. The values of the parameters τ, D and x are the same as those used in figure 7.

for given x, τ and D are similar to those for given U_0^*, τ and D. When

$$U_{0R}^* \leqslant U_0^* \leqslant U_{0\,\mathrm{COP}}^* \tag{14}$$

is satisfied, the Brownian refrigerator is operated in the optimal region. In this region, the cooling rate will increase as the COP decreases and vice versa.

Figure 9 shows that for $D = 0.3$, $\tau = 0.7$ and some different values of x, the cooling rate versus COP curves are also some loop-shaped curves, which have two important points of state: the maximum cooling rate point, $(R_{\max}^*, \mathrm{COP}_R)$ and the maximum COP point, $(R_{\mathrm{COP}}^*, \mathrm{COP}_{\max})$. The maximum cooling rate and $(\mathrm{COP})_{\max}$ increase while the COP at the maximum cooling rate and the cooling rate at the $(\mathrm{COP})_{\max}$ decrease when x is increased. This indicates
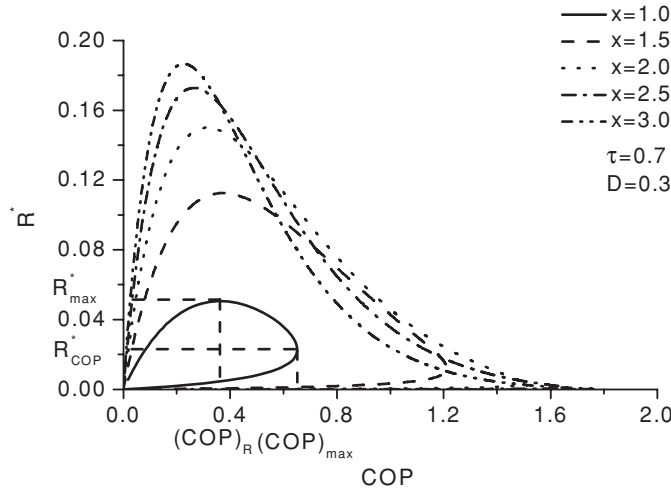
Figure 9. The curves of the cooling rate R^* varying with the COP for some given parameters, where the parameters $D = 0.3$, $\tau = 0.7$, $x = 1.0, 1.5, 2.0, 2.5$ and 3.0 are chosen.

that through the reasonable choice of the parameters F, U_0, D and τ, the Brownian refrigerator may be controlled to operate in optimal regimes.

4. The optimum regions of the external force and the barrier height of the potential

It can be seen from figures 3 and 4 that for given U_0^*, τ and D, when $x = x_{\min}$ and $x = x_{\max}$, both the COP and the cooling rate are equal to zero. Their physical meanings are very clear. When $x \leqslant x_{\min}$, F is so small that Brownian particles run along a reverse way. In this case, F becomes a load, so Brownian particles work as an engine. In $x_{\min} < x < x_{\max}$, the larger the x, the larger the $\dot{N}_+$, while the smaller the $\dot{N}_-$. When $x = x_{\max} \approx [U_0^* - 0.5(1 - \tau)]/D$, $\dot{N}_+ \gg \dot{N}_-$ and $\dot{Q}_C = 0$. Neither the cooling rate nor the COP may be obtained. Consequently, if and only if the relation

$$(k_B T_H / L) x_{\min} < F < (k_B T_H / L) x_{\max} \tag{15}$$

is satisfied, the ratchet can work as a two-reservoir refrigerator.

Similarly, it can be seen from figures 6 and 7 that for given x, τ and D, if and only if the relation

$$k_B T_H U_{0\,\min}^* < U_0 < k_B T_H U_{0\,\max}^* \tag{16}$$

is satisfied, the ratchet can work as a two-reservoir refrigerator.

5. The maximum COP and cooling rate

For the given parameters D and τ, using equation (8) and the conditions $\partial(\mathrm{COP})/\partial x = 0$ and $\partial(\mathrm{COP})/\partial U_0^* = 0$, one can find that the COP has a maximum at $x = x_{\mathrm{COPM}}$ and $U_0^* = U_{0\,\mathrm{COPM}}^*$, which are the solution of the following transcendental equations:

$$x[f_1 - 1]\{[2(U_0^* - Dx) - (1 - \tau)][1 + D(1/\tau - 1)]f_1 - 2D[f_1 - 1]\}$$
$$- \{[2(U_0^* - Dx) - (1 - \tau)]f_1 - 2(U_0^* - Dx) - (1 - \tau)\}$$
$$\times \{[x(1 + D(1/\tau - 1) + 1]f_1 - 1\} = 0 \tag{17}$$

Table 1. Optimal parameters at the maximum coefficient of performance for given D and τ.

D	τ	$(\text{COP})_{\max}$	R_M^*	$U_{0\,\text{COPM}}^*$	x_{COPM}
0.1	0.3	0.404	8.69×10^{-23}	24.91	48.29
	0.5	0.938	2.39×10^{-19}	24.95	23.38
	0.7	2.174	9.32×10^{-15}	24.97	10.57
0.5	0.3	0.385	3.82×10^{-16}	24.93	27.49
	0.5	0.916	4.28×10^{-14}	24.96	17.14
	0.7	2.149	1.28×10^{-12}	24.99	9.13
0.9	0.3	0.368	3.42×10^{-11}	24.95	19.21
	0.5	0.893	4.46×10^{-11}	25.01	13.53
	0.7	2.122	4.77×10^{-11}	25.09	7.99

Table 2. Optimal parameters at the maximum cooling rate for given D and τ.

D	τ	$R_{\max}^*$	$(\text{COP})_{\text{RM}}$	$U_{0\,\text{RM}}^*$	x_{RM}
0.1	0.3	0.0341	0.0081	4.56	39.05
	0.5	0.1115	0.0132	4.57	38.07
	0.7	0.2077	0.0194	4.58	37.29
0.5	0.3	0.0343	0.0087	17.97	34.64
	0.5	0.1116	0.0143	18.18	34.86
	0.7	0.2078	0.0210	17.51	33.32
0.9	0.3	0.0344	0.0111	24.95	27.00
	0.5	0.1117	0.0185	24.96	26.9
	0.7	0.2079	0.0261	24.97	26.8

and

$$[f_2 - 1]\{2[f_2 - 1] + 2(U_0^* - Dx)(1 - 1/\tau)f_2 - (1 - \tau)(1 - 1/\tau)f_2\}$$
$$- (1 - 1/\tau)\{[2(U_0^* - Dx) - (1 - \tau)]f_2 - 2(U_0^* - Dx) - (1 - \tau)\}f_2 = 0, \tag{18}$$

where $f_1 = e^{U_0^*(1-1/\tau)+[1+D(1/\tau-1)]x}$ and $f_2 = e^{U_0^*(1-1/\tau)+[1+D(1/\tau-1)]x}$.

Similarly, using equation (9) and the conditions $\partial R^*/\partial x = 0$ and $\partial R^*/\partial U_0^* = 0$, one can find that the cooling rate has a maximum at $x = x_{\text{RM}}$ and $U_0^* = U_{0\,\text{RM}}^*$, which are the solution of the following transcendental equations:

$$D[2(U_0^* - Dx) - (\tau + 1)]e^{-(U_0^* - Dx)/\tau}$$
$$+ \tau[2(U_0^* - Dx)(1 - D) + 2D + 1 - \tau]e^{-[U_0^* + (1-D)x]} = 0 \tag{19}$$

and

$$[(1 - \tau) - 2(U_0^* - Dx) - 2\tau]e^{-(U_0^* - Dx)/\tau} + \tau[(1 - \tau) + 2(U_0^* - Dx) - 2]e^{-[U_0^* + (1-D)x]} = 0. \tag{20}$$

Using equations (8), (9), (17) and (18), one can obtain the maximum coefficient of performance $(\text{COP})_{\max}$ and the corresponding cooling rate R_M and the optimal parameters $U_{0\,\text{COPM}}$ and x_{COPM} for given parameters D and τ, as indicated in table 1. Similarly, for given parameters D and τ, one can also obtain the maximum cooling rate $R_{\max}$ and the corresponding cooling rate $(\text{COP})_{\text{RM}}$ and the optimal parameters $U_{0\,\text{RM}}$ and x_{RM} for given parameters D and τ, as indicated in table 2. It can be seen from tables 1 and 2 that for differently given parameters

D and τ, the $(COP)_{max}$, R^*_{max} and the corresponding optimal parameters will be different. The larger the parameter D is, the smaller the $(COP)_{max}$ and x_{COPM}, while the larger the R_M and $U_{0\,COPM}$. Similarly, the larger the parameter D, the larger the R_{max}, $(COP)_{RM}$ and $U_{0\,RM}$, while the smaller the x_{RM}. This implies that the smaller the parameter D, the smaller the U_0 value at the $(COP)_{max}$ or the maximum cooling rate R_{max}, while the larger the x value at the $(COP)_{max}$ or the maximum cooling rate. In addition, the larger the temperature ratio τ, the larger the $(COP)_{MAX}$, R_M and $U_{0\,COPM}$ or R_{MAX}, $(COP)_{RM}$ and $U_{0\,RM}$, while the smaller the x_{COPM} or x_{RM}. This indicates that the smaller the temperature difference between the heat sink and the cooling space, the larger the x value at the $(COP)_{max}$ or the maximum cooling rate R_{max}, while the smaller the U_0 value at the $(COP)_{max}$ or the maximum cooling rate R_{max}. Consequently, through the choice of the parameters U^*_0, x, D and τ, Brownian micro-refrigerators may be controlled to operate in different behavior regimes.

6. Conclusions

We have established a simple model of the Brownian micro-refrigerator and shown that heat transfer can be controlled by a mechanical force. The performance characteristics of the Brownian refrigerator which consists of Brownian particles moving a sawtooth potential with an external force, where the viscous medium is alternately in contact with the hot and cold reservoirs along the space coordinate, are investigated. It is found that the heat flow $(\dot{N}^+ + \dot{N}^-)k_B(T_H - T_C)/2$ due to the change of the kinetic energy of the particles is transferred completely from the hot to the cold reservoir, the Brownian refrigerator is always irreversible and its COP cannot approach the COP of the Carnot refrigerator even in the quasi-static limit. This irreversibility leads to the loop-shaped cooling rate versus COP curves which are similar to the typical characteristics of other real refrigerators. It is also found that the influence of the external force, barrier height of the potential, asymmetry of the potential and temperature ratio of the heat reservoirs on the performance of the Brownian refrigerator is obvious. When the external force or the barrier height of the potential is determined by equation (15) or (16), the thermal motor can work as a refrigerator. The Brownian refrigerator may be controlled to operate in different regimes through the choice of the external force, barrier height of the potential, asymmetry of the potential and temperature ratio of the heat reservoirs.

Acknowledgments

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